Hale School

Mathematics Specialist

Term 1 2019

Test 2 - Functions

Name: _____

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Instructions:

- **Classpad and scientific calculators are not allowed**
- **External notes are not allowed**
- **Duration of test: 40 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**
- **This test contributes to 6% of the year (school) mark**

Question 1 **(9 marks)**

Consider the functions $f(x) = \frac{9}{x^2}$ and $g(x) = \sqrt{1-x}$

(a) Find

i) $g \circ f(x)$ (1 mark)

ii) the natural domain for $g \circ f(x)$ (2 marks)

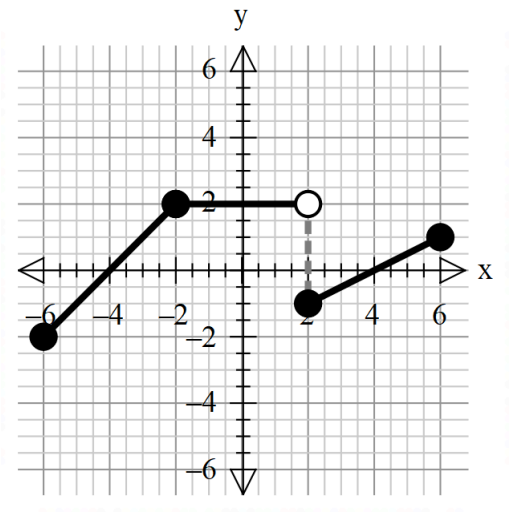
iii) the range for $g \circ f(x)$ corresponding to the domain in part ii) (2 marks)

(b) State a domain for $g \circ f(x)$ such that it is a one-to-one function. (1 mark)

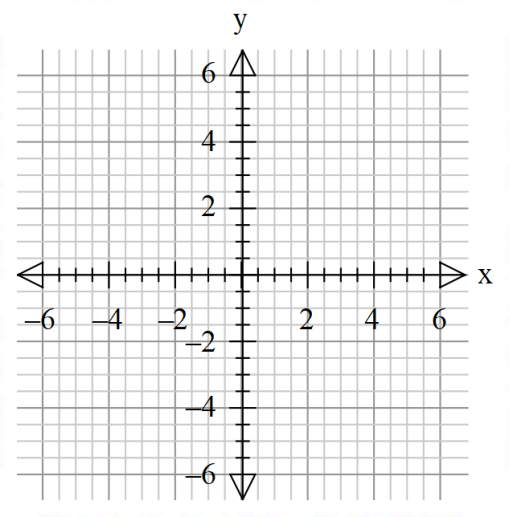
(c) For the domain in part (b), find, $(g \circ f)^{-1}(x)$, the inverse function of $g \circ f(x)$ (3 marks)

Question 2 **(5 marks)**

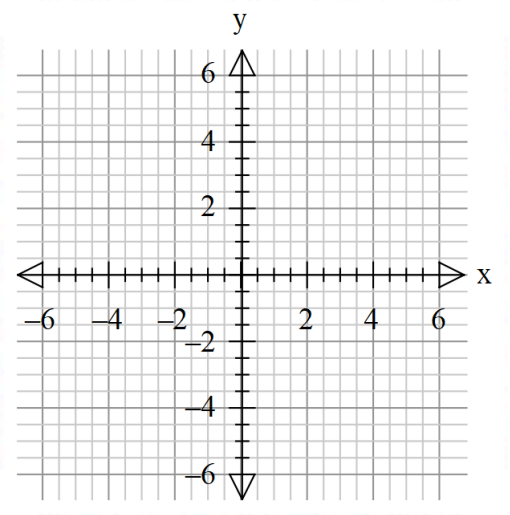
Consider the graph of the function $y = h(x)$ shown below.



(a) On the axes provided draw a graph of $y = \dfrac{1}{h(x)}$. (3 marks)



(b) On the axes provided draw a graph of $y = h(|x|)$. (2 marks)



Question 3 **(6 marks)**

Consider the function $g(x) = \dfrac{x^2 + 1}{x - 2}$.

(a) State the equation of the vertical asymptote for the graph of $y = g(x)$. (1 mark)

(b) Show algebraically, that $g(x)$ can be written in the form $ax + b + \dfrac{c}{x-2}$, stating clearly the values of a, b, and c. (3 marks)

(c) Explain clearly what writing the function in the form $g(x) = ax + b + \dfrac{c}{x-2}$ indicates about the graph of the function. (2 marks)

Question 4 **(8 marks)**

Consider the functions $p(x) = k|(x-6)(x+2)|$ and $q(x) = a - 6|x|$

Given that $y = p(x)$ and $y = q(x)$ meet when $x = 1$ and $x = 6$.

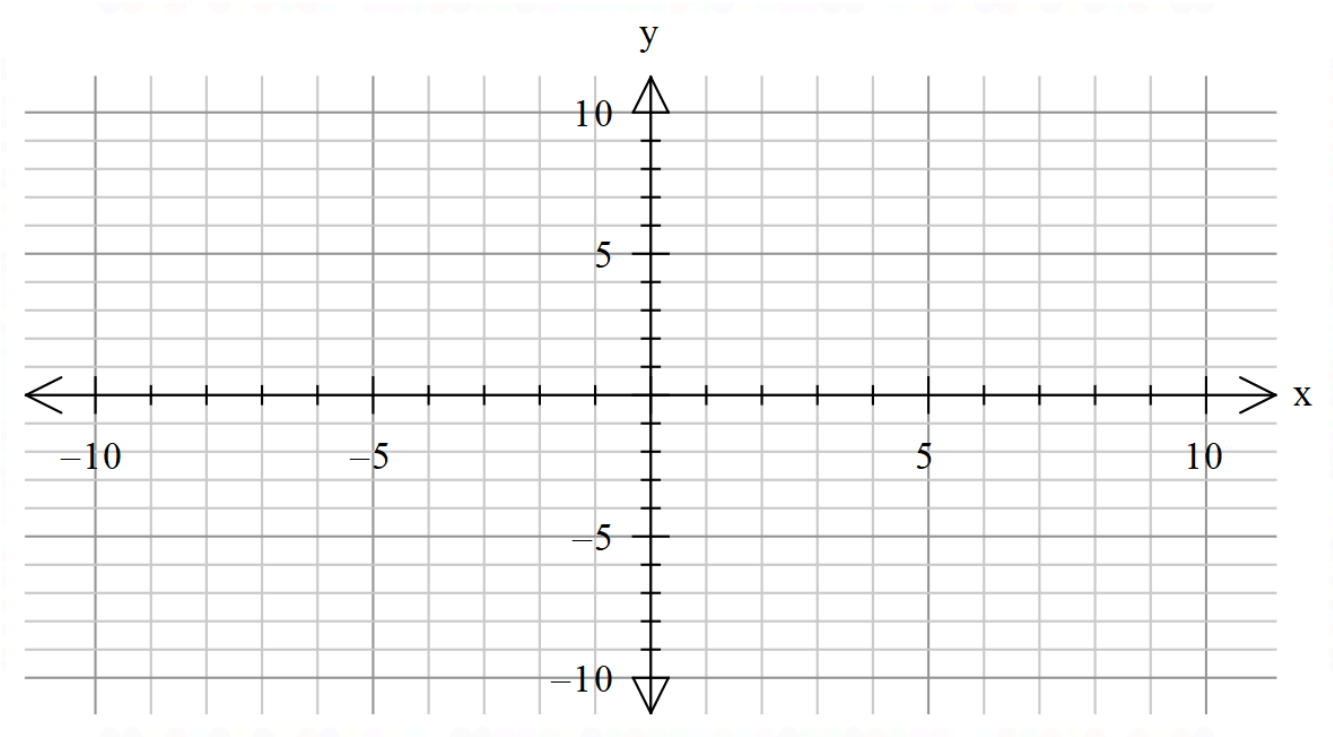
(a) Find the values of k and a. (3 marks)

(b) Find the x coordinate of any other points of intersection of the two graphs.
(3 marks)

(c) State the range of values of b for which the equation $p(x) = b$ has exactly 4 solutions.
(2 marks)

Question 5 **(5 marks)**

On the axes below draw the graph of $f(x) = \dfrac{3(x-2)(x+1)}{(x+2)(x-1)}$, showing all features.



Question 6 **(4 marks)**

Consider the function $g(x) = \dfrac{2}{x+2}$.

If $f \circ g(x) = \dfrac{x+8}{x+2}$, find $f(x)$